Quantum Foundation

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2.1. Formalism of Quantum mechanics.

- States to describe systems
- evolution
- measurement / observables

1. Basic concept: wave - particle duality

wave	particle
Interference	discrete
Superposition	Quantization
Linear Algebra	Discrete eigenvalues.

2. States in Quantum machanics.

Assumption 1: states $\Leftrightarrow$ a vector in <u>Hibert Space</u>

- vector. $|\psi\rangle$ → Inner product $\langle\psi_1|\psi_2\rangle$

$\langle\psi|$ → complex conjugate.

norm. $\langle\psi|\psi\rangle \geq 0$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Hermitian:

$A = A^\dagger$

$A^\dagger : \stackrel{(def)}{} \langle\psi_1|A|\psi_2\rangle = \langle\psi_2|A^\dagger|\psi_1\rangle^*$

$\begin{cases} \sum_i |\lambda_i\rangle\langle\lambda_i| = I \\ \langle\lambda_i|\lambda_j\rangle = \delta_{ij} \end{cases}$

<u>Spectral Decomposition</u>

$A = \sum_i \lambda_i \underline{A_i}$ — orthogonal projection (see LA 6.6).

$= \sum_i \underline{\lambda_i} |\lambda_i\rangle\langle\lambda_i|$

real — outer product

$|\psi\rangle = \sum_i |\lambda_i\rangle\langle\lambda_i|\psi\rangle$

3. Observables in Quantum mechanics.

Assumption 2: Observables $\Leftrightarrow$ a Hermitian operator. in <u>Hibert Space</u>

$|\psi\rangle \xrightarrow{A} A|\psi\rangle$

operators $\xrightarrow{\text{a chosen basis}}$ matrices.

coordinate	x	$\hat{x}$
momentum	p	$\hat{p} = -i\hbar\nabla$
Energy.	$E = E(x, p)$	$\hat{H}(\hat{x}, \hat{p}) = \frac{\hat{p}^2}{2m} + V(\hat{x})$

Hamiltonian operator.

4. Evolution in Quantum mechanics.

$|\psi(t=t_0)\rangle \longrightarrow |\psi(t=t_0+\Delta t)\rangle$

Assumption 3: Schrödinger Equation.

$i\hbar\frac{\partial|\psi\rangle}{\partial t} = \hat{H}|\psi\rangle$

if $\hat{H}$ is time - independent.

$A = \sum_i \lambda_i |\lambda_i\rangle\langle\lambda_i|$

$f(A) = \sum_i f(\lambda_i)\ |\lambda_i\rangle\langle\lambda_i|$

$-i\hat{H}t/\hbar$ ... $e^{-i\frac{E_i}{\hbar}t}$ ...

$i\hbar \frac{\partial|\psi\rangle}{\partial t} = \hat{H}|\psi\rangle$

if $\hat{H}$ is time-independent.

$|\psi(t)\rangle = e^{-i\hat{H}(t-t_0)/\hbar}|\psi(t_0)\rangle$

Evolution operator $U(t,t_0)$

$U^\dagger U = I \Leftarrow$ Unitary

norm-preserving (see LA 6.5).

$|\langle\psi(t)|\psi(t)\rangle = \langle\psi(t_0)|U^\dagger U|\psi(t_0)\rangle = \langle\psi(t_0)|\psi(t_0)\rangle$

$e^{-i\hat{H}t/\hbar}|\psi\rangle = \langle\psi_{H_i}|\psi\rangle \cdot e^{-i\frac{E_i}{\hbar}t}|\psi_{H_i}\rangle$

5. Quantum no-cloning theorem

system	A	B
initial	$|\psi\rangle \otimes$	$|0\rangle$ ← blank state.
COPY	$|\psi\rangle \otimes$	$|\psi\rangle$

Tensor Product.

$|\psi_1\rangle \otimes |\psi_2\rangle$

Proof. If this process is possible

$\Rightarrow \exists$ unitary operator U:

s.t. $U(|\psi\rangle_A \otimes |0\rangle_B) = |\psi\rangle_A \otimes |\psi\rangle_B$

This COPY should work for any $|\psi\rangle$.

$\Rightarrow |\psi_1\rangle, |\psi_2\rangle$ with. $\langle\psi_1|\psi_2\rangle \neq 1$ or 0.

$$\Rightarrow \begin{cases} U(|\psi_1\rangle_A|0\rangle_B) = |\psi_1\rangle_A|\psi_1\rangle_B \\ U(|\psi_2\rangle_A|0\rangle_B) = |\psi_2\rangle_A|\psi_2\rangle_B \end{cases} \xrightarrow{\text{(Inner product)}} {}_A\langle\psi_1|\psi_2\rangle_A \underbrace{{}_B\langle 0|0\rangle_B}_{=1} = {}_A\langle\psi_1|\psi_2\rangle_A \, {}_B\langle\psi_1|\psi_2\rangle_B$$

Contradiction!

$\Rightarrow$ Great for Security

6. Measurement in Quantum mechines.

Assumption 4. When measuring A under state $|\psi\rangle$.
the state is projected to an eigenstate $|\lambda_i\rangle$ of A with
prob. $P_i = |\langle\lambda_i|\psi\rangle|^2$ and outcome $A = \lambda_i$

Define von Neumann projectors $P_i \stackrel{\text{def}}{=} |\lambda_i\rangle\langle\lambda_i|$

$$\Rightarrow \begin{cases} P_i \geq 0 \\ P_i^2 = P_i \\ P_i P_j = \delta_{ij} P_i \\ \sum_i P_i = I \end{cases}$$

Initial $|\psi\rangle$

Final (After Measurement)

$P_i|\psi\rangle = \langle\lambda_i|\psi\rangle \cdot |\lambda_i\rangle$